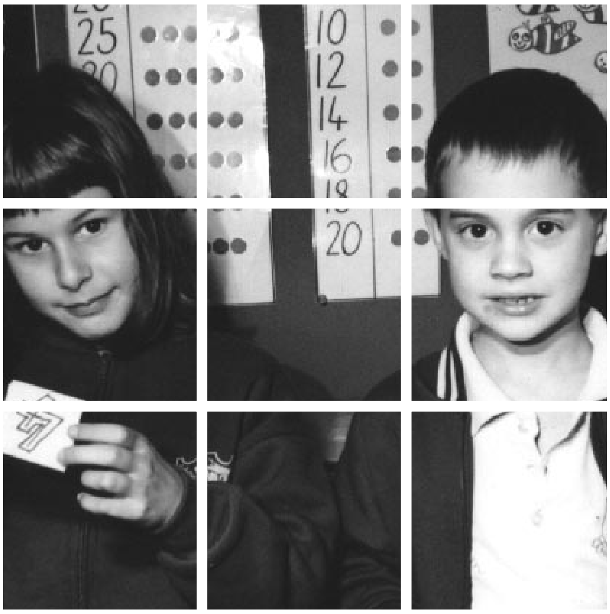
MATHEMATICS

Learning Area Statement



In mathematics, students learn to use ideas about number, space, measurement and chance, and mathematical ways of representing patterns and relationships, to describe, interpret and reason about their social and physical world. Mathematics plays a key role in the development of students' numeracy and assists learning across the curriculum.

Definition & Rationale

In the Mathematics learning area, students learn about mathematics, what it is and how it is used in making decisions and solving problems. Mathematics involves observing, representing and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves:

> *Mathematics is often defined as the science of space and number … [but] a more apt definition [is that] mathematics is the science of patterns. The mathematician seeks patterns in number, in space, in science, in computers, and in imagination. Mathematical theories explain the relations among patterns … Applications of mathematics use these patterns to 'explain' and predict natural phenomena … (Steen, L.A. (1988), "The science of patterns", Science, 240, 29, 616.)*

Mathematics can enhance our understanding of the world and the quality of our participation in society. Since it is valuable to us individually and collectively, it should be an integral part of the general education of every young person.

This statement is based on three premises:

- All students are capable of learning the mathematical ideas and skills that underpin a wide range of everyday uses and can benefit from doing so.
- All students have a right to learn mathematics in a way that enables them to see that mathematics itself makes sense, that they can make sense of mathematics, and that working mathematically can help them make sense of their world.
- For students to become confident and capable users and learners of mathematics we will need common high standards and flexible curricula which respond to students' non-standard learning needs.

Students' future personal and occupational needs will vary, as will the demands of the times. Students should, however, learn to deal readily and efficiently with commonly occurring situations that can benefit from the use of mathematics: for example, everyday decision making often involves asking questions about the 'cheapest', 'best', 'biggest', 'furthest along', 'quickest', 'most reliable' or 'most likely', and answering such questions requires facility with number, measurement and chance. Planning, planting and maintaining a garden, making garden furniture, and sewing covers for chairs all make considerable demands of spatial and measurement skills.

Students also need to be able to use their mathematics in tackling new or unfamiliar tasks. A student nurse trying to understand how the amount of medication in the bloodstream is related to the time since administration began may need to find a previously unseen formula, read symbolic expressions, ensure that the measurements to

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be used are in the right units, and rearrange the computation needed to enter it efficiently into a calculator. Activities such as making sense of a magazine article that uses the terms ‘fertility’ and ‘mortality’ to describe birth and death rates, rather than states, also call for mathematical thinking, albeit less obviously so.

Being numerate is about having the disposition and competence to use mathematics to solve practical problems outside mathematics and as a tool for learning beyond the mathematics classroom. The Mathematics Learning Area takes a major, although not sole, responsibility for the development of students’ numeracy. Students should learn to read, write and speak mathematics in a variety of contexts and forms so that they can interpret and convey mathematical ideas, interpret prose containing mathematical forms, and continue to use and learn mathematics autonomously. Whether dealing with familiar or unfamiliar tasks, they need to:

- recognise when mathematics might help;
- choose appropriate mathematics;
- decide on levels of precision and accuracy;
- do the mathematics;
- interpret the results; and
- judge the reasonableness of results and appropriateness of the methods used.

Informed numeracy involves knowing what mathematics is and isn’t, and what it can and cannot do, in order to judge and question the appropriateness of its use in particular situations. Students need to learn to ask about and question the assumptions underpinning particular uses of mathematics: for example, upon reading an advertisement that claims a bank had ‘10 million happy customers’ in the previous three months, the critical reader would say: ‘That figure doesn’t make sense. It’s half the people in Australia. Upon what assumptions were the numbers based?’ Importantly, students should also learn that mathematics cannot determine what we should or should not do in any particular circumstance. Thus, it may assist us to predict the effect on groups of individuals of introducing a new tax, but not whether we should introduce the tax – the latter is a matter for ethical and other considerations.

Many students develop strong views about mathematics during their schooling: what it is about, who it is for, and what kind of people need it and are good at it. Some are effectively excluded from some of life’s opportunities because they, and others, assume that they cannot do ‘it’. For this reason. it is essential that school mathematics be as rewarding as we can make it, that all students feel, and be, able to learn mathematics, and that students develop a positive attitude to their own continued use of it. Every student needs to develop an awareness of the nature of mathematics, how it is created, used and communicated, for what purposes, and how it both influences and is influenced by the things we believe and the values we hold.

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Mathematics Learning Area Outcomes

APPRECIATING MATHEMATICS Students appreciate the role mathematics has had, and continues to have, in their own and other communities. In particular, they:

1. Show a disposition to use mathematics to assist with understanding new situations, solving problems and making decisions, showing initiative, flexibility and persistence when working mathematically and a positive attitude to their own continued involvement in learning and doing mathematics.

2. Appreciate that mathematics has its origins in many cultures, and its forms reflect specific social and historical contexts, and understand its significance in explaining and influencing aspects of our lives.

WORKING MATHEMATICALLY Students use mathematical thinking processes and skills in interpreting and dealing with mathematical and non-mathematical situations. In particular, they:

3. Call on a repertoire of general problem solving techniques, appropriate technology and personal and collaborative management strategies when working mathematically.

4. Choose mathematical ideas and tools to fit the constraints in a practical situation, interpret and make sense of the results within the context and evaluate the appropriateness of the methods used.

5. Investigate, generalise and reason about patterns in number, space and data, explaining and justifying conclusions reached.

NUMBER Students use numbers and operations and the relationships between them efficiently and flexibly. In particular, they:

6. Read, write and understand the meaning, order and relative magnitudes of numbers, moving flexibly between equivalent forms.

7. Understand the meaning, use and connections between addition, multiplication, subtraction and division.

8. Choose and use a repertoire of mental, paper and calculator computational strategies for each operation, meeting needed degrees of accuracy and judging the reasonableness of results.

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MEASUREMENT

Students use direct and indirect measurement and estimation skills to describe, compare, evaluate, plan and construct. In particular, they:

9. Decide what needs to be measured and carry out measurements of length, capacity/volume, mass, area, time and angle to needed levels of accuracy.

10. Select, interpret and combine measurements, measurement relationships and formulae to determine other measures indirectly.

11. Make sensible direct and indirect estimates of quantities and are alert to the reasonableness of measurements and results.

CHANCE AND DATA

Students use their knowledge of chance and data handling processes in dealing with data and with situations in which uncertainty is involved. In particular, they:

12. Understand and use the everyday language of chance and make statements about how likely it is that an event will occur based on experience, experiments and analysis.

13. Plan and undertake data collection and organise, summarise and represent data for effective and valid interpretation and communication.

14. Locate, interpret, analyse and draw conclusions from data, taking into account data collection techniques and chance processes involved.

SPACE

Students describe and analyse mathematically the spatial features of objects, environments and movements. In particular, they:

15. Visualise, draw and model shapes, locations and arrangements and predict and show the effect of transformations on them.

16. Reason about shapes, transformations and arrangements to solve problems and justify solutions.

ALGEBRA

Students use algebraic symbols, diagrams and graphs to understand, to describe and to reason. In particular, they:

17. Recognise and describe the nature of the variation in situations, interpreting and using verbal, symbolic, tabular and graphical ways of representing variation.

18. Read, write and understand the meaning of symbolic expressions, moving flexibly between equivalent expressions.

19. Write equations and inequalities to describe the constraints in situations and choose and use appropriate solution strategies, interpreting solutions in the original context.

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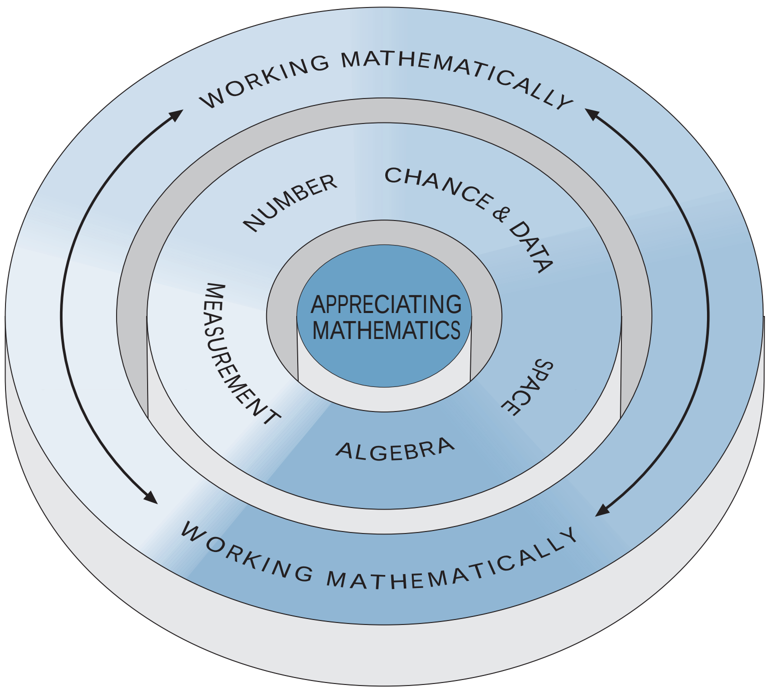
INTRODUCTION

The expected outcomes in this section provide a framework for curriculum in mathematics. These outcomes are common and comprehensive in that all students should have access to learning programs which addresses the complete set of outcomes and, in general, will be expected to achieve in relation to all of them. The outcomes are not all encompassing, however, since they will not address all of the needs of every student. There will be other outcomes which some students will be expected to achieve during their school education: for example, students undertaking specialist studies in the post-compulsory years or education support students developing independent living skills.

The outcomes are organised into seven clusters: Appreciating Mathematics, Working Mathematically, Number, Measurement, Chance and Data, Space, and Algebra. Judgements can be made about the success of students' learning in each of these clusters but, in the case of the first of the Appreciating Mathematics outcomes, the focus of judgements may be groups of students rather than individuals. The paragraphs accompanying each outcome provide elaboration and clarification of the meaning of the outcome, drawing on examples from across the years of schooling.



APPRECIATING MATHEMATICS

Students appreciate the role mathematics has had, and continues to have, in their own and other communities. In particular, they:

1 Show a disposition to use mathematics to assist with understanding new situations, solving problems and making decisions, showing initiative, flexibility and persistence when working mathematically and a positive attitude to their own continued involvement in learning and doing mathematics.

Students respond positively to their own use of mathematics and are inclined to ask, 'Will mathematics help here?', even in situations where it is not specifically required of them. For example, unprompted they count to decide whether they have enough straws for their classmates, use ideas about symmetry when classifying plants, work out their chances of winning a game with different strategies, and sketch a graph to help understand a paragraph in a textbook which describes a relationship between unemployment and immigration.

Students understand that they will continue to learn and use mathematics for all of their lives, and have confidence in their capacity to do so. They are willing to learn from others by listening to, and observing, the mathematical activity of peers, teachers and families. They appreciate, however, that working mathematically also, and often, involves finding out for themselves. They persist with mathematical tasks, pose mathematical questions, make conjectures, try alternatives and pursue different ideas. They also read mathematics to explore mathematical ideas which they have not explicitly been taught and to understand material containing mathematical ideas. Confronted with a situation in which mathematics they have not yet learned might help, they do not retreat but rather are inclined to believe that they can learn what they need or want to know.

2 Appreciate that mathematics has its origins in many cultures, and its forms reflect specific social and historical contexts, and understand its significance in explaining and influencing aspects of our lives.

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Students appreciate that mathematics is developed by people of all cultures in response to practical, aesthetic and spiritual needs, and that it is influenced by, and influences, our world views. They explain, for example, how they and others use mathematics in regulating their lives: at school we eat when the clock says 12 o'clock; in netball we have to stay within the semicircle when shooting a goal; and we are admitted to higher education if our 'score' exceeds a certain number. They also comment on how such uses of mathematics both reflect and influence the way we live and the things we value. Students might describe how the need for a form of social organisation adapted to local conditions led to Aboriginal kinship systems that can be seen as mathematical in nature. They might also describe the impact of the collection of public statistics on the daily lives and thinking of all Australians, consider how they may be used to legitimise authority structures, and analyse the ethical implications of such uses in respect of human rights and social justice.

Students understand something of the nature, power and scope of mathematical activity and respect its origins in human intuition, creativity and reason. They appreciate the intellectual leap people needed to make to invent zero, understand why people gain pleasure from number and spatial patterns, and recognise the special nature of mathematical argument. Students take a constructively critical view of the uses of mathematics, identifying situations where mathematics can enhance their own and others' lives, but also where it is misused or used to mislead or intimidate.

WORKING MATHEMATICALLY

Students use mathematical thinking processes and skills in interpreting and dealing with mathematical and non-mathematical situations. In particular, they:

3 Call on a repertoire of general problem solving techniques, appropriate technology and personal and collaborative management strategies when working mathematically.

Students draw on a range of general strategies when dealing with mathematical problems to which they have no readily available method of solution. These include such things as: act it out; guess, check and improve; look for patterns; draw a picture or make a model; solve a simpler version of the problem first; identify and attempt sub-tasks; generate and systematically list possibilities; and eliminate possibilities. They make thoughtful use of technology to enhance their mathematical work and have a range of management strategies to help them get started and keep going in individual and collaborative problem solving. They know that working independently on a problem can enable them to get a firm grasp of its features and bring their own unique perspective to its solution. But they also recognise the value of working with others, cooperating to pool ideas and welcoming, and dealing constructively with, conflicting perspectives and views.

4 Choose mathematical ideas and tools to fit the constraints in a practical situation, interpret and make sense of the results within the context and evaluate the appropriateness of the methods used.

Students recognise when mathematics may assist in dealing with a practical problem. They choose mathematical ideas, procedures and technology which suit the physical, social or ethical constraints in a situation, considering the assumptions they need to make in order to use the mathematics involved. Thus, they know that for most people multiplying the distance they can run in a minute by 60 won't give a good estimate of how far they can run in an hour, but multiplying the water lost from a dripping tap in a minute by 60 will give a good estimate of the water lost in an hour.

They consider the levels of precision and accuracy needed, make appropriate use of the results of using technology and express results in ways suited to the context, perhaps by rounding to a sensible number: for example, asked how many buses are needed to transport 397 students if each bus can hold 54, they say 8 rather than the 7.3518518 which appears on their calculator or the 7 to which the calculator answer rounds. They also judge the appropriateness of the methods used. Thus, given a problem of six hungry children and only three apples, they respond to a suggestion that the children draw 'lots' for the apples by querying whether having an equal chance of getting an apple makes this a 'fair' or 'good' solution and ask, 'Is there a better way?'

5 Investigate, generalise and reason about patterns in number, space and data, explaining and justifying conclusions reached.

Students observe regularities and differences and describe them mathematically. By identifying common features in mathematical situations, they make generalisations about numbers, space and data. Thus, they may observe that every time they combine 3 things with 9 things, they get 12 things and make the generalisation that 3 add 9 is always 12. They know that many patterns may be observed in the one situation, and generate and investigate a number of different conjectures about it. Students understand that a mathematical generalisation must be true always rather than mostly and that one exception invalidates it. They attempt to confirm or refute their own and others' generalisations and prepare arguments to convince themselves and others that a generalisation must hold in every case and not only for all the cases tried. Thus, in investigating quadrilaterals they may note that every four-sided figure which they try tessellates. This leads them to conjecture that all quadrilaterals tessellate and to search for a general argument which will convince both themselves and their peers of this. They write (and speak) mathematics clearly and precisely, expressing and explaining their generalisations verbally and with standard algebraic conventions.

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NUMBER

Students use numbers and operations and the relationships between them efficiently and flexibly. In particular, they:

6 Read, write and understand the meaning, order and relative magnitudes of numbers, moving flexibly between equivalent forms.

Students read, write, say, interpret and use numbers in common use, including whole numbers, fractions, decimals, percentages and negative numbers. They can order numbers and understand the relevance of the order: for example, they know that if they have 9 shells and 7 honky nuts, they do not have to line up the items to say whether they have more shells or nuts. They also know that cordial which is one quarter concentrate will be stronger than cordial which is one-fifth concentrate: that a library book with call number 7.52 is located after one with call number 7.513; and that a temperature of -16˚c is colder than -3˚c. They understand the relative magnitudes of numbers: for example, 9 is always 2 more than 7, '30% off' is not quite as good as 'one third off' and one million is a thousand times as big as a thousand. They choose forms of numbers helpful in particular contexts and recognise common equivalences, such as that one-fifth is the same as $^1/_5$, two-tenths, 0.2 and 20%. They interpret large and small numbers for which few visual or concrete referents are available and represent them, including using scientific notation. Their number repertoire includes irrational numbers such as π and $\sqrt{2}$ which arise in practical contexts.

7 Understand the meaning, use and connections between addition, multiplication, subtraction and division.

Students understand the meaning of addition, subtraction, multiplication and division as distinct from how to carry out the calculations associated with them. They decide which operation is needed, even when no obvious verbal cues indicate which operation is expected: for example, they understand that they can solve 'take away' and 'comparison' situations by subtracting, and write the appropriate subtraction. They recognise the need to multiply and divide in situations involving repeated addition, arrays, rates and conversions, areas, and enlargements and reductions. They know that multiplication does not always 'make bigger' or division 'smaller' and can say under what circumstances each does and does not. They also recognise and can deal with familiar and unfamiliar situations involving ratio and proportion. They understand key relationships within and between the four basic operations and use these to construct equivalent expressions, find unknown quantities and assist computation: for example, they can think of 9 as composed of 4 and 5, so that $9 = 4 + 5$, but also $9 - 4 = 5$ and $9 - 5 = 4$, and understanding such relationships enables them readily to solve problems such as $\square - 7 = 11$, or even $\square - 348 = 434$.

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8 Choose and use a repertoire of mental, paper and calculator computational strategies for each operation, meeting needed degrees of accuracy and judging the reasonableness of results.

Students are justifiably confident of their capacity to deal, correctly and efficiently, with everyday counting and computational situations. They can count a collection one-to-one, recognise skip counting in twos or threes as a more efficient way of getting the same result, and combine collections using strategies such as counting on. They know the addition facts to 10 + 10 and multiplication facts to 10 x 10, and extend these with a flexible repertoire of mental strategies for each of the four operations on whole numbers, money and simple fractions. They use written approaches as a back-up for calculations they cannot store completely ‘in the head’. These may include diagrams, jottings, standard routines, and supporting technology for students with disabilities. They understand that calculators or computers are the sensible choice for repetitive, complex or lengthy calculations and use them efficiently, correctly interpreting calculator displays. Students judge the appropriate level of accuracy, are accurate when necessary, and otherwise estimate and approximate. Unprompted, they check that the results of their computations make sense, both in terms of the numbers and operations involved and the context in which the calculation arose.

MEASUREMENT

Students use direct and indirect measurement and estimation skills to describe, compare, evaluate, plan and construct. In particular, they:

9 Decide what needs to be measured and carry out measurements of length, capacity/volume, mass, area, time and angle to needed levels of accuracy.

Students know that the same things can be compared and ordered by different attributes such as length, capacity or mass, depending upon the purpose of the measurement. They also realise that the attribute chosen will make a difference to the order. They describe their comparisons appropriately: for example, using ‘taller-shorter’, ‘wider-narrower’. They understand that we use a unit when we want to quantify ‘how big’ or ‘how much bigger’ and that generally measurements are only as accurate as the unit chosen. When choosing a unit (that is, deciding what attribute to measure and how accurate they need to be), students think about the purpose of the measurement and the closeness of the comparisons to be made. Thus, to check whether a large box will fit through a door, they recognise handspans as probably ‘good enough’ units but, if it looks like being a tight squeeze, will find a smaller unit.

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They know that standard units are not more accurate than non-standard units but they do help with record keeping and communication and are usually needed when using formulae. Students use common measuring equipment and graduated scales, such as rulers, clocks and shop and kitchen scales, choosing equipment or techniques to suit the situation. They express measurements in suitable units and use their understanding of the common metric prefixes to move flexibly between units and to judge size.

10 Select, interpret and combine measurements, measurement relationships and formulae to determine other measures indirectly.

Indirect measurement is used when direct comparison or measurement of quantities is impossible, impractical or simply tedious. Students choose and use a range of methods of indirect measurement. They may weigh a few pieces of fruit at a time and add the weights because their scales won't accommodate more than 500 g. They predict when a video will finish by taking the time now and adding on the 'length' of the film. They may also use division or averaging to find measurements more accurate than their equipment allows: for example, measuring the thickness of a ream of paper in order to calculate the thickness of one sheet. They also use formulae for finding lengths, areas and volumes; scale and similarity; Pythagoras' theorem and trigonometric ratios for finding lengths and distances in three dimensional contexts; and rates and derived measures, such as speed and density, for calculating quantities.

11 Make sensible direct and indirect estimates of quantities and are alert to the reasonableness of measurements and results.

Students have a good idea of the size of common standard units, make sensible estimates with them, and have the disposition and skills to judge the reasonableness of estimates and measurements. They know that to estimate which of two rocks has the bigger volume, looking may be sufficient, but to compare their masses, hefting will probably be needed. They have a range of benchmarks which they use in estimation: for example, they may know which of their fingers is about a centimetre wide, what a litre container of ice-cream looks like, and how heavy a kilogram of rice feels. They use these benchmarks to judge reasonableness, saying, for example, that the average height of children in their class simply cannot be 2.3 metres – they must have made a mistake. Students also reason from known and collected quantities to estimate quantities which cannot be found directly or conveniently: for example, to estimate how much food is put in the school's bins each week or how many cats there are in Western Australia.

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CHANCE AND DATA

Students use their knowledge of chance and data handling processes in dealing with data and with situations in which uncertainty is involved. In particular, they:

12 Understand and use the everyday language of chance and make statements about how likely it is that an event will occur based on experience, experiments and analysis.

Students recognise that many situations are somewhat unpredictable: for example, winning the netball, whether it will rain on the way home from school or getting a good hand in a game of cards. They make appropriate use of the everyday language of chance such as 'might', 'could', 'likely' and 'unlikely', 'certain' and 'uncertain', 'possible' and 'impossible', 'probably', 'odds', 'fifty-fifty'. They realise that situations with uncertain individual outcomes may show long-term patterns in their behaviour and that we use this to help interpret data and make predictions in order to address questions such as: 'How many mice will we have by next month?' 'What will the weather be like for our celebration in July?' 'How long will the battery last?' Students compare events, using numerical and other information to order them from those least to those most likely to happen. They know that probability is the way we quantify how likely it is that something will happen and they can interpret the probability scale from 0 to 1. They estimate probabilities from experiments and simulations, using the long-run relative frequency. They also use systematic lists, tables and tree diagrams to assist them to analyse and explain possible outcomes of simple experiments, and to calculate probabilities by analysis of equally likely events.

13 Plan and undertake data collection and organise, summarise and represent data for effective and valid interpretation and communication.

Students systematically collect, organise and record data to answer their own questions and those of others: for example, 'Which school lunch is liked best?' 'Which animal is most scary?' 'What shapes and proportions do people like best?' 'How does absenteeism from school relate to the time of year?' 'How much water is used in the school each year and for what?' 'Does having a part-time job impact on school results?'. They clarify and refine questions and plan surveys, experiments and simulations to help answer them in unbiased ways, considering both the data collection instruments and the size and nature of samples.

Students understand that (1), classification underlies the organisation of data; (2), how we classify depends upon the questions we want to answer; (3), the way the data are organised can illuminate or mask certain of their features; and (4), this influences how the data are interpreted and used: for example, one classification of sports preferences might suggest that students prefer ball games; another might suggest that balls as such are not relevant, rather that students prefer team to individual sports.

They, therefore, realise that data can be distorted accidentally or deliberately to reach inappropriate conclusions.

Students describe patterns in data and make concise but meaningful summaries using statistics to describe proportions, averages and variability. They choose and use diagrams, tables, plots and graphs which are suited to the kind of data and the purpose of the display. They consider the impact of technological change on the collection and handling of data and of the issues this raises about matters of privacy and social monitoring. They also consider ethical issues in the collection, organisation and representation of data and act responsibly in this regard.

14 Locate, interpret, analyse and draw conclusions from data, taking into account data collection techniques and chance processes involved.

Students interpret and report on their own data (about age and height, pet preferences, pocket money, fitness levels, attitudes to smoking, a weather simulation) and data taken from a variety of secondary sources (magazines and newspapers, sports records, a history database, a health report on bicycle accidents, the Internet, a set of farm records). They locate and use databases about Australia and Australians, such as those available through the ABS. They distinguish between census and sample data and understand that considerable care needs to be taken in selecting samples and in forming conclusions about the whole group from sample data. In this, they realise that the uncertainty involved in drawing conclusions from data is the essence of the connection between ‘chance’ and ‘data’.

Students know that when making use of data they should question their quality and credibility, and the way in which they are organised and represented before evaluating the conclusions drawn by others. They recognise that good-quality data and a knowledge of chance processes can help them assess risks, form opinions and make decisions such as, for example, whether or not to immunise children. They also know, however, that while mathematics and data can each contribute to our decision making, they cannot determine what we should or should not do in any particular circumstance, the latter being matters of personal and/or community judgement.

SPACE

Students describe and analyse mathematically the spatial features of objects, environments and movements. In particular, they:

15 Visualise, draw and model shapes, locations and arrangements and predict and show the effect of transformations on them.

Students recognise shapes in different orientations, sections and diagrams and visualise the effect of transforming them in particular ways. Thus, they can predict the different shapes of the faces produced when a carrot is cut in different ways and how a shape will appear when reflected in a mirror. They visualise how an object will look if the viewer walks around it through a quarter turn and whether a particular net will fold up to make a planned container. In drawing shapes and spaces, they use the common mathematical techniques available to show 3D things in 2D forms: for example, using ellipses to represent circles and dots to indicate lines which cannot be seen but must be there; showing things further away as smaller; and parallel lines coming closer together in the distance. They make suitable choices from among different forms of representation for particular situations. They also construct full-size and scale models of places and structures: for example, they may use junk equipment to build a model of their local shopping centre, produce a careful scale drawing of their classroom, or plan a net to make a container to hold three tennis balls.

Students visualise and sketch paths, regions and arrangements to meet specifications, perhaps by showing their route to school, planning a visit to the zoo or an optimum route for a tour of country towns by a rock group, sketching the region covered by a special sprinkler system, or interpreting a kinship network. They know that different communities, past and present, representing location and arrangements differently, can identify mathematical features in these representations, and use conventional mathematical forms of representations, including coordinates and networks.

16 Reason about shapes, transformations and arrangements to solve problems and justify solutions.

Students use conventional names (square, cylinder) and criteria (curved, parallel) to describe and analyse 2D and 3D shapes. They make general statements about the properties of shapes and relationships between them, including similarity and congruence: for example, they know that for a shape to be a square it must have four equal angles as well as four equal sides; that if the diagonals of a four-sided polygon cross at their mid-point, it must be a rectangle; and that prisms always have two opposite faces which are the same shape and size. Further, they apply this knowledge to link the shape and structure of objects to their function and hence to the problems of design: for example, they explain why some shapes are not widely used as floor tiles, why milk cartons are the shape they are; why some shapes are suitable for building purposes and others not, and how the properties of the diagonals of a rectangle enable builders to ensure that the corners of their structures are right-angled.

Students describe the effect of transformations on shape, size, orientation and arrangement. Thus, they recognise the symmetries associated with reflection, rotation and translation in contexts such as totems, clothing, crystal growth and map projections. They enlarge and reduce shapes to specification and understand how the similarity properties of enlargement and reduction enable us to produce such technology as overhead projectors and photocopiers and mathematical techniques such as those of trigonometry.

ALGEBRA

Students use algebraic symbols, diagrams and graphs to understand, to describe and to reason. In particular, they:

17 Recognise and describe the nature of the variation in situations, interpreting and using verbal, symbolic, tabular and graphical ways of representing variation.

Students use words, tables, symbols and graphs to describe and understand relationships. They sketch graphs based on word descriptions of situations, tables of data or an algebraic expressions. They may, for example, explain in words how their hunger level varies through a typical day and draw an informal sketch graph to show it; produce a scattergraph by plotting a point for each student in their class which shows TV watching time against reading time; or sketch a graph showing the relationship between the varying population levels in a herd and the amount of water available based on the formula which relates them. They interpret graphs by attending to global and local features. Thus, they may inspect a friend's graph about hunger levels and conclude that the overall pattern of hunger levels is quite similar to their own but that the friend's family must eat later in the evening, suggest that TV watching time and reading time appear unrelated, and comment that the water available to the herd is decreasing at an increasing rate due to population growth.

Students understand that variables (price, height, population) can be related to each other in different ways but that certain kinds of relationships occur frequently and are therefore worthy of particular study. The types of relationships they study include linear, exponential, reciprocal, quadratic and periodic. Students recognise that it is possible to tell what type of relationship exists between two variables simply by looking at the algebraic rule or formula that relates them or a graph of the relationship. They also inspect sets of data to decide whether they can be modelled by one of the familiar families of functions.

18 Read, write and understand the meaning of symbolic expressions, moving flexibly between equivalent expressions.

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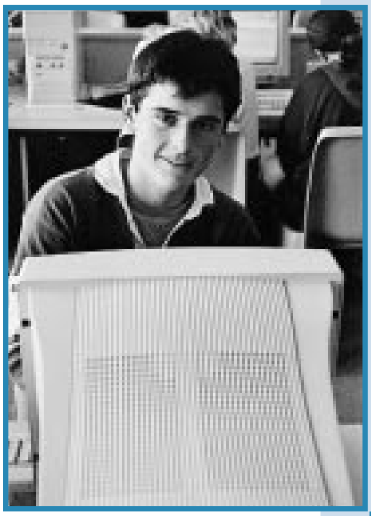
Students understand that algebraic expressions (e.g. $3d^2 + 5$) can be used to represent some aspect of reality and use them to express generalisations such as identities and rules for generating patterns. They read, write and follow rules expressed symbolically, including a wide range of practical formulae which may be comprised completely of symbols or mixtures of words and symbols such as often appear in the home or workplace. They know the basic conventions of algebraic notation that determine how expressions are built up. They can manipulate (rearrange) expressions by making use of notational conventions, the distributive property of multiplication over addition and inverses of addition and multiplication. In doing this, they understand that it is possible to operate with algebraic expressions independently of the contexts in which they arose. They understand that the purposes for rearranging symbolic expressions (including formulae) include assisting with computation, recognising types of variation and sketching graphs. Rearrangement is also often needed to show that two rules are equivalent and to solve equations.

19 Write equations and inequalities to describe the constraints in situations and choose and use appropriate solution strategies, interpreting solutions in the original context.

Students understand that equations and inequalities represent the constraints in particular situations and that 'to solve' an equation or inequality means to find all the values of the variable(s) that fulfil the constraint: for example, given information about the effect of the cost of concert tickets on the numbers of people likely to attend, they represent the constraints mathematically by writing an equation and solving the equation to find the ticket price that maximises profit. Students recognise, however, that we can use algebra to work out the price at which maximum profit is achieved, but not to decide whether profit is an appropriate criterion for setting the price – that decision will reflect a range of non-mathematical factors, including the values they hold.

Students set up their own equations and inequalities and realise that they can be solved by a range of strategies that may not relate directly to the original situation. Their repertoire of solution strategies includes guess, check and improve approaches; finding approximate solutions from graphs; and some analytic techniques, particularly for linear equations. They choose from among these strategies to suit the circumstances. They also have the mathematical understandings and technical skills needed to make sensible use of calculators and computers in solving equations.



SCOPE

The Scope of the Curriculum

The purpose of this section is to describe the suggested emphasis of curricula at four broad phases of education. It is not intended to prescribe a specific learning program. Indeed, the outcomes described within this Framework are superordinate to the specifics of any particular curriculum provided in schools and a number of different ways of addressing the outcomes might be developed.

EARLY CHILDHOOD
(typically kindergarten to year 3)

For many young children, their personal world centres on their family network and their mathematical thinking will reflect this: for example, in some Aboriginal communities, children's mastery of directional and spatial relationships is taken as a sign of intellectual prowess in much the same way that counting skills are in some other Australian communities. The spatial strengths of children should be applauded, encouraged and used as starting points for mathematical ideas, as should their number strengths.

Appreciating Mathematics

The curriculum of the early childhood years should assist children to recognise the mathematics inherent within the activities of their home communities and to take pleasure and confidence in it. Activities should also be structured to enable children to develop mathematical ideas with a wide variety of familiar materials and through problems which have meaning for them.

MATHS

Working Mathematically

Children should learn to pose mathematical questions about their world and to seek ways of answering such questions for themselves and in collaboration with their peers. While success is important, children should be challenged mathematically, encouraged to try out new ideas and to persist with tasks for increasing periods of time. Superficial success or success on personally easy tasks can put children at considerable risk in their future learning.

A feature of these years is that children build their mathematical ideas on their personal knowledge of the world. They may investigate a range of 'problem stories' involving small numbers in which both the problem and the solution strategy already make sense in their own experience. When told a story about a dog which lost some pups, for example, they might represent the story with plastic dogs and work out how many are left or how many are lost. Representing many such 'stories' over time in materials, pictures and 'the mind's eye', and making links between them, leads to the mathematical idea of subtraction. The observation of such patterns or regularities in mathematical situations provides basic building blocks for children's understanding of the structure of number and space.

Initially, children will work mostly with repeating and counting patterns but their capacity to deal with patterns should develop considerably during this period. From the earliest years, they should identify, copy and continue a variety of patterns, follow provided rules, create their own mathematical patterns, and find ways of describing the rules which produce them.

Number

Children should be learning to read, write and say whole numbers and use them to count, order and combine. Thus, they should learn the sequence of number names and how to use the sequence to determine 'how many' there are in a collection and to make a collection of a given size. Considerable work with counting is likely to be needed before children fully understand that any collection has only one count and that you can tell from the count alone which has more (that is, a collection of 24 objects always has one more than a collection of 23).

Young children need extensive experience of physically partitioning quantities so that they learn to think of numbers as compositions of other numbers, for example, 7 is composed of 1 with 6 or 2 with 5 or 3 with 4, and realise that all whole numbers can be thought of in this way. They should investigate realistic situations and fictional stories in which all four operations are embedded, representing story problems with a range of materials and pictures.

During these years, they should be assisted to memorise basic addition and subtraction facts. They should also be using a range of mental strategies for problems requiring the addition and subtraction of small numbers and extending their range with physical materials, pictures, calculators and some informal paper-and-pencil strategies. The curriculum should provide children with experience in splitting quantities into 'fair shares' and begin to develop the link between sharing and the language of fractions.

Measurement

Most children come to school with intuitive notions of 'more', 'less', 'equal amounts' and 'big and small'. Curricula should help them focus on attributes of objects and events such as length, mass, capacity, area and time and to compare and order by particular attributes. Children will require a great deal of experience to enable them to develop comparative language such as 'heavier' - 'lighter' and 'tall' - 'short', particularly since many of the subtleties are culturally specific.

Many measurement experiences will not require counting but rather the direct comparison of quantities. When children do 'count units', perhaps to decide how many cups fill the bowl, they will often think of the question literally as 'how many fit' and will not connect the count to how big the bowl is. Learning experiences should help children to see that using a unit repeatedly to match an object gives a measure of the size of the object. They then should see why the unit must be 'packed in' without overlaps and gaps.

In estimating, children should learn that they can look at a book to estimate how many matches will fit along it but must hold it in order to estimate how heavy it is. They answer such questions as 'Which feels heavier, the book or the pencil box?' and 'How many crayons are about as heavy as the book?' They learn to use language such as 'almost', 'about' and 'just under' and 'between', and begin to develop a feel for the size of centimetres and metres. Young children should also be developing their intuitive feeling for scale as a basis for work in later years: for example, they may play games requiring them to match component parts according to a rough scale ('This chair is for the baby bear').

Chance and Data

Young children should investigate actions and events which involve unpredictability and refine their use of everyday language of chance such as 'will' - 'won't' - 'might', 'possible-impossible', 'certain' - 'uncertain'. They should carry out experiments which involve chance processes (e.g. selecting a jelly bean from a bowl with their eyes shut) and examine the outcomes with discussions and predictions referring to the range of possibilities ('I could get a red or a green but a black is impossible').

Children should be assisted to classify consistently and to classify things in different ways: for example, collecting information on 'the most popular kinds of toys' or on 'the most hard-wearing kinds of toys' might require different classifications of toys. They should record and represent data which arise from practical and problem-solving activities in all of the mathematics strands and all learning areas. Children should begin to construct graphs based mostly on one-to-one correspondence. Thus, to compare how many children have short-sleeves with how many have long-sleeves, the two groups of children could line up to act as a concrete graph, pin up self-portraits to make a pictorial graph, or make a block graph by using a token for each child.

Space

Children should explore their environment and objects within it, using a variety of materials to make and arrange things to model their imaginary or real world. They should be assisted to focus on the role of the component objects in the whole construction, learning to ask if components will stack, fit together, balance, tip over or roll. They should investigate the connection between shape and function: for example, considering why bicycle wheels are round by experimenting with various shapes as 'wheels'. They should handle objects, note the 2D shapes in them (e.g. squares, rectangles, circles), observe their properties and learn to describe them in the everyday language of shape, making statements such as 'This box has eight corners. Its sides are all squares'.

Young children should carry out changes to the shape, size or position of objects and observe the effects of these changes, saying, for example, 'When I turned the square jigsaw piece around it still fitted into the same space'. This should lead to an informal understanding of different transformations. Basic mathematical ideas about location and arrangement should be developed as children arrange and rearrange familiar objects – from toys to furniture to the contents of their tray – both freely and by following oral directions. They should also give and follow directions to find their way around an environment, including a computer environment.

Algebra

In the early childhood years, children observe patterns and represent and describe these regularities in mathematical ways, providing the basic building blocks for algebra. Algebraic thinking is also involved when they take 4 grapes, eat them and, upon noting that there are 5 left, can say how many there were to start, thinking 'I take 4 from a number and have 5 left, what is the number?' Such experiences form the basis of the idea of finding an 'unknown quantity', leading in later years to setting up and solving equations. A child may correctly solve $\square - 4 = 5$ and ten other like it by trial and error or by recognising the subtraction fact and gain important number practice. It is important, however, that teachers help children go beyond this and focus on general relationships between operations, so that over time they come to see that if $\square - 4 = 5$, then $\square = 5 + 4$, because of the relationship between addition and subtraction. Knowing this relationship makes the problem, $\square - 74 = 65$ easy to solve (given a calculator).

MIDDLE CHILDHOOD YEARS (typically years 3 to 7)

Typically, the personal world of children in the middle to upper primary years has expanded to include their cohort. They regard themselves as members of, and align themselves with, particular 'collectives' such as their peer group, teams, clubs, class or school. Their ways of learning mathematics will reflect this. They work collaboratively and will often enjoy learning from and with each other by engaging in mathematical discussion.

Appreciating Mathematics

Problems should often relate to children's immediate physical or social world, although they may also come from within a computer micro-world or fiction or mathematics itself. The important thing is that problems attract and involve children and that they have some ownership over the problem and its solution.

Working Mathematically

Long periods of self-sustained mathematical activity should not be expected, but children, working individually and in groups, should be encouraged to persist with problems and to ask other questions about the situation. While their execution may at times let them down, they should be expected to take care with and check their work. Children are increasingly able to think of concepts such as 'multiplication', 'six' and 'triangle' as mathematical objects in their own right. Hence, it makes sense to them to say such things as 'To work out how many weeks in 5 years I need to multiply 52 by 5'. They may use physical materials or sketches to help compute 52 by 5, but they do not necessarily have to represent the original problem situation. They understand that one idea, like multiplication or triangle, can apply to many different 'real' situations.

The curriculum should emphasise not only building mathematical ideas from a rich variety of familiar contexts but also the investigation of mathematical ideas and relationships as such. Children should be learning to make conjectures and test them by trying a range of cases and searching for exceptions. Thus, a boy may tell his partner his conjectured rule, and she may help him test it by asking: 'Does it work for squares? Does it work for....?'

Number

Developing and generalising children's place value concepts should be emphasised during these years, so that they develop a sense of the large whole numbers and of decimal numbers. Children should be learning to read, write and use different representations of numbers: whole numbers, common fractions, decimal fractions and percentages. The meaning of decimals and their relative order and size should be a focus of experiences and should relate to contexts such as finding library books, comparing measurements and reading scales. The common meanings attached to fractions should be developing as children represent the 'part to whole' notion with a wide range of discrete and continuous materials.

Children should be learning that one mathematical operation can apply to apparently different situations: for example, the division operation is appropriate for sharing problems (e.g. '52 cards are shared with 4 people. How many cards each?') and for grouping problems (e.g. '52 cards, 4 cards for each person. How many people?'). In each case, 52 ÷ 4 is the correct operation and the same sequence of calculator keystrokes is used.

Children should expected to make conscious choices from among operations to apply in a given situation. Using concrete materials, diagrams and calculators, they should work out the basic products (10 x 10), record their findings in a range of ways and use conventional notation. They should then be helped to memorise these basic facts in ways that make them available flexibly rather then by chanting through the relevant 'table'. Through practical problem-solving situations, it should become clear to children that they need to calculate with numbers larger than those that can be dealt with by remembering number facts and they should investigate methods for carrying out those calculations. Their mental computation should develop through discussion, comparison and reflection on alternative strategies and varied practice. They should be learning to use calculators and paper-and-pencil as backup for calculations which they cannot carry out completely in the head.

Measurement

Measurement work should be practical and children should be assisted to make sensible choices about which qualities should be measured for the task at hand, which units to use, and which measuring tool is suitable. They should use units to measure a range of different attributes in a range of different circumstances: for example, they should find the areas of shapes which cannot be neatly covered in unit squares by covering the shape with units and part-units and finding ways to combine the part-units. In doing this, children should be helped to develop the idea that a unit is a size or amount rather than a particular shape or object and to make generalisations about units such as that 'the smaller the unit, the more it takes to match a particular thing.'

Children should be developing a feel for the size of millimetres, centimetres and (metres and the associated square and cubic measures), litres, kilograms and seconds and minutes, and use these in making reasonable estimates. Efforts to improve estimates should be made explicit and children should learn strategies for improving estimates. The curriculum should include the use of a range of graduated measuring scales and the relationships between units represented by the metric prefixes of milli, centi and kilo.

Children should investigate relationships used for indirect measurement concluding, for example, that the area of a rectangle made up of squares can be found by multiplying the number of squares along the top by the number down the side. Some may generalise further that the area of a rectangle is the length multiplied by the breadth. Children should be learning to use scale factors and gaining some experience of the effect of scaling on the lengths, areas and volumes of shapes.

Chance and Data

Children should collect, represent and interpret data in order to answer questions of interest to them. They should plan and carry out investigations which involve chance processes (tossing coins, selecting different-coloured lollies from a container, estimating the number of snails in the school garden) and discuss and compare the results of their experiments, noting that different results are likely in repeats. In doing so, they should be learning to order familiar and imaginary events informally from most likely to least likely.

In conducting surveys, children should work collaboratively to clarify what type of information they need to collect ('What do we mean by 'most popular pet' or 'head size'? and what people, things or events are to be surveyed. This should include drafting questions, testing different versions of them for clarity and bias, and redrafting to improve their usefulness. Children should be beginning to learn that data can be classified, organised, summarised and displayed in a variety of ways and that the choices depend upon the questions being asked of the data, the type of data and the audience. The teacher can assist by introducing new methods of representation, with a repertoire that includes fractions and averages, tables, plots and graphs and databases.

During these years, children should begin to consider whether it is reasonable to generalise from their data: for example, they may decide to use the head sizes of all those in year 6 in their school to make predictions about head sizes for other 11-year-old children but decide not to generalise to much younger or older people.

Space

Children should investigate the features of objects in the environment, including their shape and the effect on them of changes in shape, size or position. Experiences should be designed to enable them to visualise and represent shapes in various orientations and sections, movements, and paths and locations. Children should make spatial patterns and investigate various symmetries and tessellations. They should predict the effects of changes they make to the shape, size or position of figures and objects and check by experimenting.

The ability to interpret and produce drawing of three-dimensional shapes develops slowly and children will need considerable experience with these. They should also investigate the relationship between three-dimensional shapes and their two-dimensional nets, leading to the capacity to sketch, plan and make models. Children should sort and classify shapes and movements according to spatial criteria and interpret spatial language and use it for themselves. They should be learning to recognise and describe the distinguishing features of various classes of shapes.

Children should also be expanding their repertoire of ways of representing and describing location to include grids and distances and directions. They should learn to relate direction and angle of turning to compass directions and use a magnetic compass to determine simple directions.

Algebra

Children should work with a variety of numerical and spatial patterns and find ways to explain what is general in them. They could make this sequence: |_| |_|_| |_|_|_| with matchsticks and note that 'You need 4 matches for the first square and add 3 for each additional square' or 'Start with 1 match and add 3 for each square'. In order to answer the question 'How does the number of matches vary as the number of squares varies?' children need to describe the pattern rule in a general form, such as 'To find the number of matches, you multiply the number of squares by 3 and then add one'.

Notions about how variation in one quantity is related to variation in another may also be developed through measurement (e.g. by comparing the diameter and circumference of many circular lids) and through daily experience of relationships such as that between the time of day and typical level of hunger. The relationships can be described in words, orally or written, and also graphically using both informal sketch graphs and those based on actual measurements. Children should be beginning to learn that relationship graphs help us get a holistic sense of how one quantity varies as another does.

Finally, a computer program which provides many questions such as $\square - 17 = 13$ can encourage children to develop a general strategy for finding the unknown quantity; cases such as $\square - 236 = 317$ can 'force' a strategy, since trial and error will be onerous. Solving one such example is not algebra: the algebraic thinking occurs when children recognise that there is something general here; that is, 'After a while I saw that I just needed to add the two numbers together and that must give the missing number'.

EARLY ADOLESCENCE
(typically years 7 to 10)

In the early adolescent years, students continue to align strongly with their peer groups. They are concerned with understanding their physical and social world but tend to focus on how it affects them personally and how they will find their place within it. As a result, for many adolescents success with mathematics is not its own reward, and neither is pleasing their parents or teachers.

Appreciating Mathematics

Students want to know how the mathematics they are learning will help them outside the mathematics classroom: at home or in the workplace. It is also closely connected to how good they think they are at mathematics and whether they believe that they are capable of making sense of mathematics. Consequently, the curriculum should seek to challenge and extend all students in mathematics but within a supportive environment for learning.

Working Mathematically

Students should be encouraged to persevere and to produce work to a quality and standard appropriate for the purpose and audience. Having worked on problems individually or collaboratively, students should reflect upon and discuss successful and unsuccessful mathematical strategies and ideas. They should also prepare some oral or written summaries, explanations and reports in which they describe and justify their mathematical processes and conclusions. While an overly pedantic attention to the technical language of mathematics is unlikely to be productive for most students during these years, reading, writing and talking mathematics can support learning and students should be assisted to develop their mathematical literacy.

In the early adolescent years, students are increasingly able to use mathematics to help make sense of the world, as distinct from using their understanding of the world to make sense of mathematics. Their widening experience of the community and other learning areas increases the range of situations to which they can apply mathematics: for example, they may draw on their understanding of the effect of scaling on area and volume to understand why babies dehydrate more quickly than adults. Various mathematical ideas are coming together, so that, in dealing with scale, students should be encouraged to draw upon and integrate ideas from number, space, measurement and algebra.

Students should also investigate purely mathematical ideas and relationships, gaining some experience with the cycles of conjecture, explanation and justification which typify pattern finding and problem solving.

Number

The curriculum should enable students to improve their capacity to represent numbers in a variety of ways and move flexibly between representations. The understanding of decimal place value should continue to be a focus of learning experiences for many students in these years. This is necessary for such everyday purposes as ordering numbers, reading scales and measuring small quantities, the important ideas being those of order and relative magnitude. Both of these become more difficult when dealing with very small and very large numbers for which concrete or visual referents are not available.

Students should be learning to apply number operations to a wide range of problem situations, developing the skills necessary to select operations and procedures and judge the reasonableness of results. They need to maintain and consolidate their techniques for mental arithmetic, estimation, calculator use and paper-and-pencil work so that they become confident of their capacity to deal with everyday computational situations correctly and efficiently. Some will also extend the types of numbers on which they can operate to include addition and subtraction of negative numbers which arise in realistic settings. Students in the early adolescent years are approaching the time when they will begin to earn money of their own and take responsibility for managing their own finances. Consequently, social and commercial arithmetic become increasingly relevant. The use of computing and calculating technology should be assumed.

Measurement

Students should be improving their measurement skills and their ability to estimate quantities. They should become proficient with commonly-used measuring equipment, develop a good feel for the size of various standard units and become competent at estimating in standard units. They should be learning that all measurement is approximate and that efficient measurement requires a sensible choice of unit; in some situations, the most accurate unit possible may be chosen but in others a rough measure may be the best choice. Students should carry out practical tasks involving measurement, They should, individually and collaboratively, plan, make judgements about which measurements to make, organise and carry out the measurements, and decide whether the results are of the right magnitude.

Students should be developing a range of sensible methods of indirect measurement. These include mensuration formulae, Pythagoras' theorem, rates and differences, similarity and scale. An important idea which should develop from the investigation of right triangles is that similar right triangles have equal corresponding ratios – the basic principle of right triangle trigonometry.

Chance and Data

Students should learn to estimate probabilities experimentally and through the analysis of simple sample spaces. They should work collaboratively and with teacher support to set up and carry out simulations.

Practical investigations should be undertaken which involve all of the facets of data handling. Students may plan and execute surveys about student opinions on such matters as environmental issues, tastes in music or what sports they want the school to offer, or collect census-type data from peers, investigating, for example, how many students have computers at home. The activities should include careful consideration of procedures for choosing samples and designing and trialing questionnaires, the comparative advantages of different methods of organising and representing data (including tables, databases, plots and graphs, and summary statistics) and the difficulties which arise at the interpretation stage, which can be used to inform later projects.

Students should be learning to interpret various representations of data including means, measures of variability and association, line plots, histograms, stem-and-leaf plots, box plots, scatter plots, and lines of best fit; understand the conditions under which their use is appropriate; and compare and select from different possible representations of the same data. Calculators are a necessary tool in this. Students should be gaining experience which will, over time, enable them to distinguish between a population and a sample, informally draw inferences from data collected by themselves and others, construct convincing arguments based on such data, and evaluate arguments.

Space

Experiences should be practical and exploratory although many students will also benefit from a more analytic study of geometry. All students should examine two-dimensional and three-dimensional geometric shapes, investigating and describing relationships between classes of shapes (e.g. all squares are rhombuses and all squares are also rectangles, but not all rhombuses and rectangles are squares). They should analyse the properties of various shapes and apply this knowledge to problems: for example, they may develop and justify an approach to ensuring that bases for a softball game are always correctly placed.

Students should visualise, demonstrate and describe the effect of reflections, rotations, translations and enlargements on shape, size, orientation and arrangement and recognise and produce associated symmetries. They should identify and explain the use of these transformations in designs such as floor tiles, Escher woodcuts and Computer Aided Design (CAD); devices such as wheels, sewing machines and projectors; and in natural things such as crystals and shells.

Students should improve their proficiency at drawing figures and constructing objects. They should build full-sized and scale models of a range of shapes which fit specifications. They should also use and compare several different conventions for representing three-dimensional objects in two-dimensions: for example, plans and elevations, isometric drawings, perspective drawings, orthogonal drawings, contour maps and Mercator and other geographic maps.

Students should learn to use conventional geometric language and techniques to show routes, paths and regions and represent locations and arrangements in networks and other diagrams.

Algebra

Students should have extensive experience in observing patterns and relationships among quantities and representing them symbolically and graphically. The study of functions should begin with a sampling of relationships familiar to students. Students should use sketch graphs to model relationships drawn from their own daily experiences, such as their mood at different times of the day, and from their understanding of other forms of variation, such as the speed of a car at various stages of a race. Their sketches should reflect the difference between discrete and continuous data and between situations which are essentially deterministic (e.g. the relationship between the radius and circumference of circles) and those which involve an element of chance (e.g. the relationship between body weight and age in children). A function grapher should enable students to develop an intuitive grasp of the general shapes of particular kinds of functions (including linear, quadratic, exponential, reciprocal and periodic) fairly rapidly and to be able to visualise the effects of translation and reflection on these functions.

Students should learn to manipulate simple algebraic expressions which occur in meaningful contexts: for example, they may produce different expressions for the general term in a matchstick sequence and then rearrange the expressions to show that they describe the same relationship: that is, they are equivalent. Students should be developing facility with notational conventions and properties such as additive and multiplicative inverses and the distributive property of multiplication over addition in order to rearrange expressions and solve equations. From spatial or numerical investigations, they should establish and come to recognise common identities such as $a^2 – b^2 = (a – b)(a + b)$ and apply these in various contexts.

Students should formulate equations and inequalities from a range of numerical, spatial and measurement contexts and develop a repertoire of ways to solve them including ‘guess, check and improve’ and graphical methods, with some analytic methods for dealing with at least linear equations. Some students may also learn to solve pairs of simultaneous equations and quadratic equations analytically. They should informally shade regions on graphs to represent various constraints: for example, given a constraint such as ‘the floor of the house will be a rectangle and have an area of between 9 and 16 squares’, they shade a region on a graph to show all possible room dimensions.

LATE ADOLESCENCE/ YOUNG ADULTHOOD

(typically years 10 to 12)

Students in the late adolescent and young adult years will vary in the level of their achievements in mathematics, personal interest in the subject, and the extent to which they value it. Their personal goals and anticipated educational and vocational directions and destinations will also vary. Some will have chosen mathematics solely as a means to an end, others are interested in mathematics itself.

Appreciating Mathematics

The curriculum should reflect the wide range of students’ current concerns and interests and enable them to develop mathematical knowledge which they can connect both to their personal aspirations and/or to their social and community concerns.

Working Mathematically

Many students will need support if they are to sustain themselves in mathematical situations. Particular attention to the reading and writing skills needed for autonomous learning of mathematics may be required.

Students are likely to regard mathematics as a body of knowledge that can be applied to the world. They should be expected to choose and use appropriate mathematical ideas and techniques by careful consideration of the circumstances and to explain and justify their choices. Given appropriate learning experiences, they generally understand that mathematics provides assumption-based models of the world and that the same physical or social situation may be modelled in different ways, sometimes with consequences in terms of predictions: for example, while we know that banks actually calculate interest on mortgages discretely (that is, daily, monthly or quarterly) we may approximate the situation using a exponential function which is continuous and, within certain limits, get very accurate predictions, but we must also be careful not to go beyond the limits.

Students should undertake projects which have their origins in the physical or social world, some of which require them to formulate and/or test mathematical models, although the mathematical sophistication of the models they use will vary considerably. The study of patterns should continue to underpin mathematical work in these years. In particular, students should observe common patterns underlying apparently-different situations: for example, recognising the exponential function in paper folding, growth and decay and interest charge problems, enabling them to further integrate and link their mathematical ideas.

Number

Many students are in the transition between the learning environment of the classroom and the world of work. In the former, part-marks are often given to encourage students and to reward them for what they do know. In the latter, accuracy may often be vital and ‘a good try’ simply not good enough. For such tasks, reliability is of prime importance and ‘part-marks for method’ may make little sense. Therefore, students should be expected to recognise situations in which absolute accuracy is essential and to ensure that they achieve it. Students should also study error propagation, becoming particularly conscious of the potential for errors to arise in the use of calculators and computers and for the effect to be cumulative. They should plan calculations, often involving the rearrangement of expressions, to increase accuracy of computation.

All students need to understand the implications that debt and the cost of borrowing money have for them. While they do not all need to understand the technical language and mathematical principles which underlie personal and commercial financial transactions, they should experience making decisions based on financial calculations in situations which are personally relevant.

Measurement

While basic concepts and skills should have been established by these years, most students will continue to benefit from experience in choosing and making measurements, providing measurements in a suitable form, describing the effect of errors when combining measures, and giving reasonable explanations of methods and results. These skills may be developed through the study of particular areas of application such as health, surveying, meteorology, sport, horticulture and physics, or in the context of modelling problems such as those in design, economics, mechanics, stock control or environmental studies. In either case, students should engage in practically-based activities which may range from designing, costing and marking out a garden sprinkler system to determining the best shape for a carton of 750 ml cans or making a study of the nautilus shell.

While most students should understand that any measurement on a continuous scale involves error, older students may find the idea of error or tolerance in measurements of more practical interest. Students should consider the importance of relative error and investigate the effect of errors when measurements are combined: for example, in finding volume or using a conversion formula. Calculators make this accessible for all students. Many of the measurements students need to obtain will require the use of indirect measurement techniques. Activities should lead to an expansion of these techniques and involve rates, circular measure and the use of similarity relationships including the use of trigonometric ratios in two and three dimensions.

Chance and Data

Major emphases of the curricula in these years should be on making sensible judgements about the quality of data which are to be the basis for decision making and about the confidence to be placed in inferences drawn from data; learning how to design surveys and experiments; and recognising common statistical fallacies. Students should use both analytical and experimental approaches to probability and learn that probabilistic models, whether theoretical or experimental, make assumptions about the behaviour of phenomena. How well these probabilities predict 'real' behaviour is, therefore, essentially an empirical question.

Students should design, conduct, analyse and report on statistical investigations, taking into account the general processes for choosing, making and interpreting measurements which apply equally to the collection of statistical data. They should informally consider statistical, practical, ethical and economic reasons which may determine the choice of a sample or the design of an experiment, and the consequences of that choice for the confidence which can be placed on the conclusions drawn.

Students should be entering and accessing information in databases and exploring data from a variety of sources, such as science, economics, public health or market research, in order to address matters of immediate and future importance to them: for example, the gender segregation of the labour market or youth unemployment. They may also consider the social impact of chance-based models with regard to such matters as safeguarding the quality and reliability of products, opinion polls, gambling and, of course, admission to higher education.

Space

All students should learn to apply geometric concepts and results to practical problems. Thus, using computer packages, they may formulate and test hypotheses in three dimensions and apply what they have learned to complex design tasks. They should learn about applications of geometry in fields other than mathematics (e.g. scaling used in the biological sciences to identify limiting factors on the growth of various organisms), even if at times the technical demands of the applications go beyond what they are expected to produce independently.

Many applications of mathematics depend upon the capacity to visualise and represent objects and transformations of those objects. The development of these skills should be explicitly addressed. 'Thought experiments', in which they imagine, for example, the effect of two spheres coming across each other's paths and moving through each other, can be followed up by the inspection of computer-generated images of these dynamic processes.

Students should also recognise that networks can be used in situations where measurements such as length or angle can be ignored. Hence, they may be used when we cannot or do not need to make assumptions about such matters as straightness, parallelism, congruence or similarity. Many problems related to optimisation of job allocation, traffic flow and location are accessible on the basis of simple network techniques and students should experience some of these applications.

Algebra

The emphasis should continue to be on students observing patterns and relationships among quantities and representing them symbolically and graphically. During the early adolescent years, students should have studied a range of discrete and continuous functions and some will have made a more detailed study of families of linear, quadratic, reciprocal, exponential and periodic functions. The repertoire of many older students, however, will be limited and they should have the opportunity to extend that repertoire, although the particular functions they use will vary.

The use and interpretation of formulae can be considerably enhanced by an understanding of algebra and, for many students, this will be its major application. Students should learn to read and use unfamiliar formulae, including those which combine words and symbols and those which are completely symbolic. They should learn that a formula is not simply an instruction: it also tells about the nature of the relationships involved. Thus, inspecting the formulas for simple and compound interest respectively, and recognising one as linear and the other as exponential, tells one 'immediately' how each will grow over time.

Students should experience some of the range of practical situations in which we need to manipulate and rearrange algebraic expressions: for example, to make computation easier, to simplify entry of a formula in a spreadsheet, or to show that two apparently-different mathematical formulations of a situation are actually the same. They should also develop their repertoire of analytic, graphical and computational strategies for solving equations using appropriate technology.

Learning, Teaching & Assessment

The purpose of this section is to elaborate the principles about learning, teaching and assessment which have informed this Curriculum Framework. It is intended to be illustrative rather than comprehensive.

LEARNING AND TEACHING

For students to become effective learners of mathematics, they must be actively engaged, and want and be able to take on the challenge, persistent effort and risks involved. This is most likely to occur when the student personally experiences an environment that is supportive but mathematically challenging and when processes that enhance sustained and robust learning are promoted.

■ *Opportunity to learn*

Learning experiences should enable students to engage with, observe and practise the actual ideas, processes, products and values which are expected of them.

Students should practise, (that is, 'do') mathematics, but doing mathematics involves much more than the repetition of facts and procedures; it also involves working mathematically across all the strands. If students are to learn to deal flexibly with fractions in both routine and non-routine ways, select the most suitable approach for a situation, try alternatives, or adapt procedures to different situations, they will need to see these processes modelled by others, and engage in them for themselves. This will require that they understand the concepts about fractions which underpin procedures and processes. Thus, they will need to engage in activities which focus on the meaning of fractions, rather than simply procedures for dealing with them.

A limited number of skills should be automated and some repetition will help with this. Students should, however, practise the skill which they actually need to develop: for example, to develop mental addition skills, students need spaced and varied practice with a repertoire of alternative addition strategies and an emphasis on choosing strategies suited to the particular numbers. Repetitive exercises on a written vertical addition algorithm are unlikely to improve mental addition, indeed, they are more likely to interfere with it.

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■ *Connection and challenge*

Learning experiences should connect with students' existing knowledge, skills and values while extending and challenging their current ways of thinking and acting.

Learners' interpretations of new mathematical experiences depend on what they already know and understand: for example, often the first experience children have of the decimal point is in the context of money and measure. As a result, many develop the idea that the decimal point is a separator of two whole numbers (the dollars and the cents; the metres and the centimetres), a reasonable 'first estimate' of the meaning of the decimal point, even though inadequate in the longer term. Good teaching will help students to clarify or bring to the surface their understanding of decimals in order to extend, refine or discard unhelpful ideas and construct a more sophisticated understanding of the meaning of the decimal point.

If additional ideas about decimals are introduced without connecting to and challenging their existing ideas, students may continue to hold on to their earlier understanding, even when they are apparently successful with the new material. Thus, students may have learned that the first column to the right of the decimal point is the tenths and the second position is the hundredths and yet, underneath this, still think of the decimal point as separating two whole numbers. This can lead them to a number of different errors. For example, they may expect a book with a Dewey Decimal Number 3.125 to come after one with Number 36.65, continue the sequence 1.2, 1.4, 1.6, 1.8 … with 1.10, and, round an answer of $3.125 to $4.25. Dealing with each of these errors separately is likely to be unproductive and inefficient – the underlying misconception should be addressed.

A mathematical challenge to this understanding could occur when the student finds that the book is not where he expects it to be, punches the sequence into a calculator by repeatedly adding 0.2 and find the calculator goes from 1.8 to 2.0, or estimates that $25 ÷ 8 is very close to $3 and cannot be more than $4. To learn from the challenge or conflict, the student must recognise it, see errors as a useful source of feedback, believe that mathematics is supposed to make sense and that inconsistent answers need to be thought about, and respond by trying to find some way of dealing with the inconsistency.

Action and reflection

Learning experiences should be meaningful and encourage both action and reflection on the part of the learner.

Mathematical learning is most successful when students actively engage in making sense of new information and ideas. If students face mathematical situations that are not inherently meaningful, then they are forced to conclude either that mathematics does not make sense or that they themselves are incapable of making that sense. Providing students with isolated facts and procedures which they are expected simply to imitate and remember, or with partial explanations of concepts disconnected from their other mathematical ideas forces them to resort to learning strategies based on the passive reception of mathematical concepts and processes and the role imitation of procedures. The result is likely to be short term storage that needs to be topped up regularly, rather than effective long-term learning.

How students respond to a task and what they learn from it depends upon their conception of the task. Students should be helped to distinguish between activities which provide drill to increase accuracy and efficiency on already-learned procedure, and activities which develop understanding of mathematical concepts and processes or require them to apply mathematics in new ways. For the former, they should expect to be able to get started almost immediately and, otherwise seek help.

For the latter, persistance, thoughtfulness, struggle and reflection are expected as they work out what to do for themselves. Students should be taught to reflect upon what did and did not work and why, and how it connects to other mathematics.

■ *Motivation and purpose*

Learning experiences should be motivating and their purpose clear to the student.

Mathematics is often promoted to students as an investment in the future and for some students this is sufficient motivation to keep them working at it. For others, however, this is not persuasive and the mathematics provided in school must provide its own motivation if such students are to continue to participate actively. All students, however, should have opportunities to experience the satisfaction and pleasure that mathematics can bring. Students should use mathematics in decision making and problem solving about situations that are interesting in their own right and not simply because they demonstrate some mathematical idea.

Effective learning requires that students feel able to risk making mistakes without fear of the consequences. This means that the purpose of activities and hence expectations must be clear to students so that they know which risks are reasonable. Learning about families of functions may involve students in predicting what a set of related graphs will look like. Students should not be inhibited from conjecturing or making quick sketches for fear that they will be judged negatively if their early tries don't work or are messy. Later, however, they would be expected to correctly predict graph shapes and sketch them with care.

■ *Inclusivity and difference*

Learning experiences should respect and accommodate differences between learners.

Linguistic, cultural, gender and class differences between students are often regarded as adequate explanations for differences in mathematical achievement. This Framework starts from the premise, however, that a common cause of many students' failure to learn mathematics in a sustainable and robust way is an inadequate match between the curriculum and the experiences and understandings of students. For example, many children come to school able to count collections of 6 or 7 by pointing and saying the number names in order, but they do not have the visual memory to recognise 6 or 7 at a glance. Others (and this may be more common in some Aboriginal and Torres Strait Islander communities), may recognise 6 or 7 objects at a glance without being able to say the number names in order. In each case, the students' existing knowledge should be recognised and used as the starting point for further learning. In each case, it should be extended to include the complementary knowledge, with the new knowledge being linked to, building on and challenging the students' existing ideas and strategies, so that over time they develop mathematical understandings which are both commonly accepted and over which they feel some ownership.

Independence and collaboration

Learning experiences should encourage students to learn both from, and with, others as well as independently.

Collaborative learning can enhance mathematical learning in a number of ways. Firstly, by working together and pooling ideas, students can develop ideas and solve problems which may be inaccessible to them individually. Secondly, students' command of mathematical ideas and mathematical language is likely to improve when they try to describe, explain or justify. Thirdly, discussion is one of the ways students come to understand that others may not interpret things in the same way or share their point of view. Finding that a friend is not convinced that all quadrilaterals will tessellate may motivate the student to rethink, clarify and refine his or her ideas and ways of talking about them, and to develop better arguments to justify the claim. The sceptical friend may be provoked to think about what she or he knows in fresh ways or to work on what she or he doesn't know, perhaps coming to see that quadrilaterals can be thought of as two triangles with a common edge and that this provides a way of showing how all quadrilaterals tile.

Working individually is also important in mathematics. It should enable students to ensure a personal grasp of concepts, processes and procedures. In turn, they should develop confidence in their capacity to do mathematics for themselves. Students will need help to develop strategies for getting started and persevering in mathematical situations.

Supportive environment

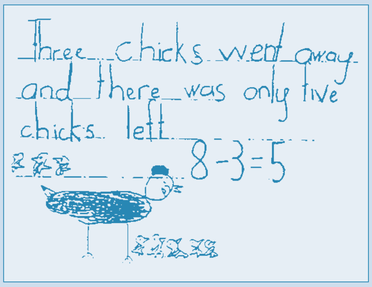
The school and classroom setting should be safe and conducive to effective learning.

High levels of unproductive anxiety about school mathematics are common, even among students who achieve well. This anxiety is associated with certain beliefs about mathematics. Firstly, there is a widespread and deep-seated view that you either have 'a mathematical mind' or you don't and that those who do are quick thinkers, can respond instantly to tasks and recognise an appropriate solution strategy immediately. Secondly, mathematics is seen to be either right or wrong and the feeling of exposure associated with being wrong is quite debilitating for many students. In order to cope and avoid feelings of failure, some students 'don't try' and, as a result, progress very little as they proceed through school.

The belief that students' confidence in mathematics will increase if they have continued success sometimes tempts us to explain exactly what to do, reducing the risk of error, and expecting and accepting less high-level thinking of certain students. Many learners become debilitated by continued success on personally easy tasks, however, and increasingly avoid situations in which they might make mistakes or be found out, and become less able to take the risks needed for higher-level learning. It is important that students learn to flounder in a constructive way rather than to avoid all stress and struggle, and that the mathematics classroom be typified by challenge within a supportive learning climate.

Understand the meaning, use and connections between addition, multiplication, subtraction and division …

[a]



[b] *Write or tell a story problem that would require this calculation:*
379 – 280

[c] *Match the word stories to the calculations.*

- *1678 + 92*
- *1678 – 92*
- *1678 x 92*
- *1678 ÷ 92*

(1) Out of a batch of 1678 eggs, 92 were too small to be sold. How many saleable eggs were there?

(2) 92 people took a 9-day package tour costing $1678 per person. How much did they pay altogether for the tour?

(3) The type of aeroplane that was used to lift 1678 stranded townspeople to safety could carry 92 passengers. How many 'lifts' were needed to transport them?

(4) There were 1678 students who applied for a job at the Brisbane Show. Only 92 were successful. How many students were turned away?

(5) 1678 tickets were sold for a concert in one day. 92 more tickets were sold the next day. How many tickets were sold in all?

ASSESSMENT

The assessment principles in the Overarching Statement apply to all outcomes in the Mathematics Learning Area. However, they are illustrated with examples relating to just one outcome from the Number strand. **Students understand the meaning, use and connections between addition, multiplication, subtraction and division.** This could be thought of as 'understand operations'. A wider range of assessment activities would be needed to deal with all aspects of the Number strand and all strands. The number in paragraphs links the principles to each of the illustrated examples.

■ *Valid*

Assessment should provide valid information on the actual ideas, processes, products and values which are expected of students.

Each of the assessment tasks and situations illustrated on this page provides information on some aspect of 'understand operations'. Together, they deal with knowing what an operation means, its properties and when to use it, and understanding relationships between operations. Examples (a) and (b) indicate students' understanding of the meaning of the subtraction sign, while example (c) deals with whether they know when to use particular operations, example (g) with the relationship between addition and subtraction, and examples (h) and (i) with whether they choose to use the operations for practical tasks. Even for this one outcome, however, more assessment information is needed to ensure that all four operations and rates are addressed.

Often, we assess a different outcome than the one intended, perhaps assessing whether students can compute rather than whether they understand the operations. Example (c) focuses explicitly on choosing the operation and does not confuse this with the ability to do the calculation, an important but different outcome. Whether or not a task validly assesses a particular outcome depends upon what the student does, not what the writer of the task intended. Example (d) is intended to assess understanding of the effect of multiplication and division on numbers. If a student were to ignore the directions and calculate 246 x 1.3 in order to insert the appropriate sign, then the answer could not be used as evidence on 'understand operations'. Other evidence would have to be sought in order to decide whether the student understood, in general the effect of multiplication and division on numbers.

Educative

Assessment should make a positive contribution to students' learning.

Each of the assessment activities here could form part of the ongoing learning activities of the classroom. In generating as many as they can for example (e), students were forced to look for patterns and invent strategies. They also self corrected, as shown in the work sample provided. Tasks such as example (d) and (g) should be the subject of debate within the classroom, with students offering their justifications to the class. Example (h) and (i) provide students with opportunities to try out their knowledge of operations in practical situations and example (j) acts as a diagnostic activity, providing feedback to teachers and learners about students' current thinking.

Explicit

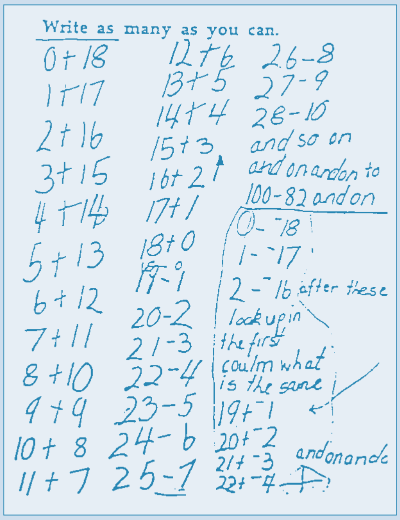
Assessment criteria should be explicit so that the basis for judgements is clear and public.

Students will be assisted in their learning if they are clear about what is expected of them. In example (d), making clear that it is what multiplying and dividing do in general to numbers, rather than calculation, that is of interest, will help students to focus their learning. Even where questions are more open, it is generally helpful to indicate what is expected. Some teachers might react positively to the pattern generation and understanding of the relationship between related subtractions shown in example (e). Others might respond less positively, considering the number sentences too alike and not creative, and look for a greater variety of types. If such judgements are to be made, then students should be aware in advance ('Think of as many different types as you can' or 'See whether patterns can help you'). If the task is truly open ended, then each of these types of student response should be acknowledged positively.

[d] *Without doing any calculation, decide whether the left hand side is bigger than (>) or less than (<) or equal to (=) the right hand side:*

246 x 1.3	. . .246
98 x 1.001	. . .98
920 ÷ 0.8	. . .920
920 x 0.8	. . .920
246 ÷ 1.3	. . .246
98 ÷ 1.001	. . .98

[e] *Presented orally: As I walked past a classroom I heard a girl say '18'. The teacher said, 'that's right'. What could the question have been? Write down as many as you can.*



[f] *Sarah, on holiday in Bali in 1998, needed 100 000 rupiah to buy some clothes. She saw this sign outside a money change shop. How many Australian dollars did she need to change?*

	Buy rupiah	Sell rupiah
US Dollar	12 091	9 923
UK Pound	19 796	16 345
Australian Dollar	7477	6087
French Franc	2018	1673
Japan Yen	87	72

[g] *Shane and Gerry were asked to solve a problem. Shane said, 'I think we have to work out this*

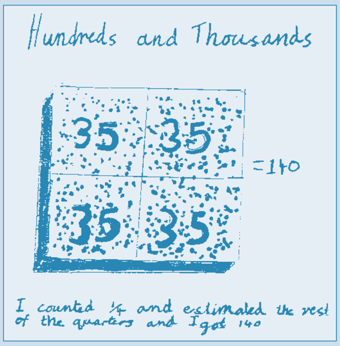
☐ - 18 = 19.'

Gerry said, 'No, I think it is 18 + 19 = ☐ and we can't both be right.'

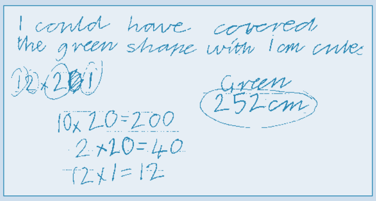
Shane thought they could both be right?

What do you think? Explain why you think that.

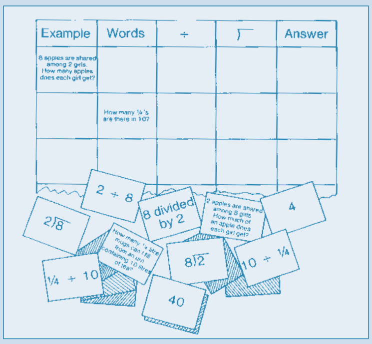
[h]



[i] *Children were handed two five sided green shapes and told to imagine they were paddocks and decide which would give more grass. No measurements or any other information was provided but a range of materials were available.*



[j] *Think and then put the cards in the right places.*



■ *Fair*

Assessment should be demonstrably fair to all students and not discriminate on grounds that are irrelevant to the achievement of the outcome.

All students should be judged on the same outcomes but this does not mean that all students should be assessed with the same tasks. Fairness often means that different tasks are needed. A student with reading difficulties would find it difficult to demonstrate the capacity to choose an operation with example (c) unless it were to be presented orally. Students for whom the story lines in example (c) are familiar and make sense are more likely to be able to recognise what a sensible answer would be and hence to choose the appropriate operation. A student recently arrived from Indonesia or one who visits Bali regularly may know that you always get more rupiah than dollars and use this in order to decide whether to multiply or divide in example (f). A student who has not had such experience has a much more difficult task to perform and must really understand the nature of an exchange rate. For these students the same task would assess different things. While every assessment cannot be equally familiar or fair to every student, the range of tasks and situations should be such that students are treated equally fairly.

■ *Comprehensive*

Judgements on student progress should be based on multiple kinds and sources of evidence.

If assessment is to be valid and fair, a wide range of assessment tasks will be needed for any particular outcome. Examples (c) and (d), although unconventional in content, are each standard short-answer 'test' type questions and it would be possible to ask students quite a number of such questions in a short time and to mark them quickly. For example (d), observation of students at work and discussion with them about their strategy for answering the questions may be needed to decide whether they were able to answer without doing the calculation. Example (h) arose from a project in Chance and Data, in which the student freely chose to use multiplication to solve a practical problem. In example (i), Sarah was given an open-ended measurement task in which the teacher did not anticipate the strategy chosen. All of these kinds and sources of evidence should 'converge' in order to provide convincing evidence on the achievement of outcome.

Links Across the Curriculum

Mathematics is integral in the education of young people. The Mathematics learning area, either directly or indirectly, contributes to each of the Overarching outcomes. In addition, through numeracy, it provides learning skills which contribute the achievement of the outcomes for most of the learning areas of the curriculum.

LINKS TO THE OUTCOMES IN THE OVERARCHING STATEMENT

Students' achievement of the outcomes in Working Mathematically, Number, Measurement, Chance and Data, Space, and Algebra will enable them to deal with quantitative and spatial ideas; to collect, display, analyse and interpret data; and to describe and reason about patterns and relationships. Thus, the Mathematics learning area takes a major – although certainly not sole – responsibility for ensuring that students:

- select, integrate and apply numerical and spatial concepts and techniques;
- recognise when and what information is needed, locate and obtain it from a range of sources and evaluate, use and share it with others; and
- describe and reason about patterns, structures and relationships in order to understand, interpret, justify and make predictions.

The Mathematics learning area supports and reinforces the literacy work of the school by setting expectations and providing feedback and support to students that is consistent with current thinking in literacy. It also takes a major responsibility for assisting students to learn to read, write, listen to and talk about mathematics, and to develop the range of special symbols, vocabulary and diagrammatic representations that mathematics contributes to language. In these ways, the Mathematics learning area makes a direct contribution to enabling students to:

- use language to understand, develop and convey ideas and information and interact with others.

Mathematics plays a central role in the generation of technology generally and has, itself, changed significantly as a result of the impact of computing technologies. Through the achievement of a number of the Mathematics learning area outcomes, students:

- select, use and adapt technologies.

Through the Mathematics learning area, students come to appreciate the way in which mathematics is embedded in the very fabric of our own and other societies. Their understanding of the cultural and intellectual significance of mathematical activity, and how it influences what we are and might be, enhances the extent to which they:

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- understand their cultural, geographic and historical contexts and have the knowledge, skills and values necessary for active participation in Australian life;
- interact with people and cultures other than their own and are equipped to contribute to the global community; and
- participate in creative activity of their own and understand and engage with the artistic, cultural and intellectual work of others.

The outcomes in Appreciating Mathematics and Working Mathematically address the mathematical attitudes, appreciations, and individual and collaborative work habits that enable students to be critical, creative and confident users of mathematics. With the provision of a supportive environment for learning mathematics, appropriate mathematical challenge and teaching processes that foster autonomous learning in mathematics, this means that students:

- visualise consequences, think laterally, recognise opportunity and potential and are prepared to test options;
- are self-motivated and confident in their approach to learning and are able to work individually and collaboratively; and
- recognise that each person has the right to feel valued and be safe and, in this regard, understand their rights and obligations and behave responsibly.

Finally, the Mathematics learning area makes an indirect contribution to several Overarching outcomes – and hence to other learning areas – as a result of its responsibilities for numeracy. Numeracy generally benefits learning in most parts of the school curriculum, while being innumerate can inhibit and even prevent student learning elsewhere. In particular, being numerate can significantly enhance students' capacity to:

- understand and appreciate the physical, biological and technological world and have the knowledge and skills to make decisions in relation to it;
- understand their cultural, geographic and historical contexts and have the knowledge, skills and values necessary for active participation in Australian life;
- value and implement practices that promote personal growth and well-being; and
- participate in creative activity of their own and understand and engage with the artistic, cultural and intellectual work of others.

NUMERACY ACROSS ALL LEARNING AREAS

Numerate behaviour can be thought of as the disposition and competence to use mathematics in the service of endeavours other than mathematics. Numeracy is linked to 'what mathematics you know', but it also involves the skills, thinking processes and attitudes needed to choose and use mathematics outside mathematics. In this sense, numeracy is about practical knowledge that has its origins and importance in the physical or social world rather than in the conceptual field of mathematics itself.

The major responsibility for developing students' numeracy lies with the Mathematics learning area. At all levels, teachers of mathematics should help students learn to use their mathematics to solve practical problems and as a tool for learning beyond the mathematics classroom. Teachers of mathematics should also take responsibility for teaching students to read and write in situations that involve mathematical ideas, notations and visual forms.

However, the development of numeracy involves more than mathematics classrooms alone can provide, since its achievement requires working mathematically in a range of different settings. Indeed, school mathematics is unable to fully capture all that is numeracy simply because the mathematics is in mathematics. In order for them to be 'numerate', students must learn to connect the mathematics from situation to situation – including across the school curriculum and beyond. Learning areas other than mathematics can, therefore, contribute to the enhancement of students' numeracy by:

- providing rich contexts in which students can use their mathematics;
- expecting students to use their mathematics in other learning areas; and
- maintaining common and challenging standards.

In The Arts learning aArea, for example, students might consider what shapes they and others find pleasing. In doing so they are likely to use and also to enrich the language of shape and number developed in mathematics. Mathematics lessons might build upon this work on form, using the golden ratio as a starting point for considering ratios in general. Mathematics lessons could also involve students in trying to find out whether people actually do prefer particular shapes and forms, thus developing important ideas about data collection and handling and about the value and applicability of their mathematical work.

The Society and Environment and the Health and Physical Education learning areas may each call upon and enrich number, measurement and data skills when students investigate such matters as water wastage or rubbish generation and disposal in the school yard, their own fitness levels or adolescent health. The Society and Environment and Mathematics learning areas may also provide complementary work which develops an understanding of how three-dimensional space can be represented in two dimensions – as in various map projections – but never without some distortion, so that interpreting a two-dimensional representation requires an understanding of the features of the three-dimensional space that are and are not preserved in the two-dimensional representation.

The Technology and Enterprise learning area will also both draw upon and enhance students' understanding of the representation of three-dimensional space in two dimensions. Indeed, the design and production of models involved in achieving the outcomes of Technology and Enterprise will draw extensively upon number, measurement and space concepts and skills, and should also considerably enhance students' learning in these areas by providing a broader range of contexts and experiences than could be provided in mathematics alone.

The Science, Society and Environment and Mathematics learning areas each contribute to students' understanding of how time may be represented sequentially or in a linear fashion and of how the tracking and measurement of time relates to cyclical or periodic phenomena. These learning areas, together with the Languages Other than English learning area, can help students to understand that people may conceive of time differently and in culturally-specific ways. Thus, students can come to understand that the application of particular mathematical ideas to the measurement and recording of time is both influenced by and influences how we think about and experience time in our daily lives and in our myths and legends.

The English learning area provides the language foundations essential for the learning of mathematics and the development of numeracy. Equally, developing students' capacities to draw on a wide range of mathematical ideas in their reading and viewing generally is a major contribution of numeracy to English. English and mathematics together provide the basic information skills involved in reading the daily newspaper or a telephone book, and in preparing reports.

Thus, within each learning area, two questions should be asked:

- How can the learning area enhance students' numeracy?
- How can numeracy contribute to enhanced outcomes in the learning area?

Answering these two question will require teachers to collaborate in developing common interpretations of numeracy and strategies to assist their students to use mathematics across and beyond the school curriculum.

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